

EE105 – Fall 2014

Microelectronic Devices and Circuits

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Lecture12-Small Signal Model-BJT

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Introduction to Amplifiers

- **Amplifiers:** transistors biased in the flat-part of the i-v curves
 - BJT: forward-active region
 - MOSFET: saturation region
- In these regions, transistors can provide high voltage, current and power gains
- Bias is provided to stabilize the operating point (the Q-Point) in the desired region of operation
- Q-point also determines
 - Small-signal parameters of transistor
 - Voltage gain, input resistance, output resistance
 - Maximum input and output signal amplitudes
 - Power consumption

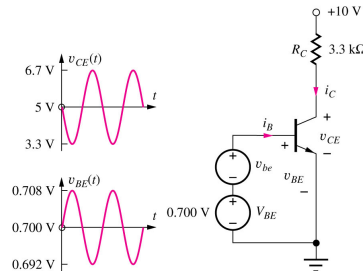


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Transistor Amplifiers BJT Amplifier Concept



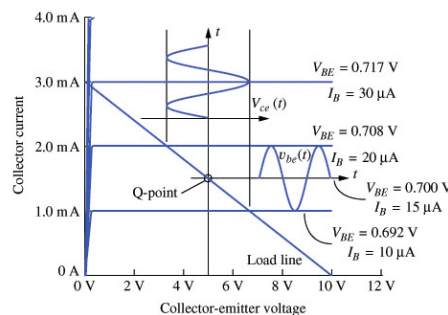
The BJT is biased in the active region by dc voltage source V_{BE} . e.g., Q-point is set at $(I_C, V_{CE}) = (1.5 \text{ mA}, 5 \text{ V})$ with $I_B = 15 \mu\text{A}$ ($\beta_F = 100$)

Total base-emitter voltage is: $v_{BE} = V_{BE} + v_{be}$

Collector-emitter voltage is: $v_{CE} = V_{CC} - i_C R_C$ This is the load line equation.



Transistor Amplifiers BJT Amplifier (cont.)



8 mV peak change in v_{BE} gives 5 mA change in i_B and 0.5 mA change in i_C .

0.5 mA change in i_C produces a 1.65 V change in v_{CE} .

If changes in operating currents and voltages are small enough, then i_C and v_{CE} waveforms are undistorted replicas of the input signal.

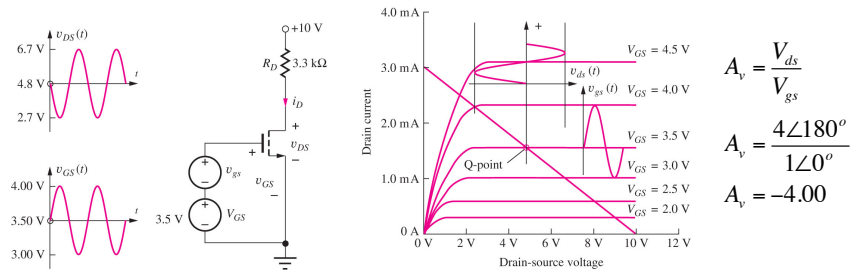
A small voltage change at the base causes a large voltage change at collector. Voltage gain is given by:

$$A_v = \frac{V_{ce}}{V_{be}} = \frac{1.65 \angle 180^\circ}{0.008 \angle 0^\circ} = 206 \angle 180^\circ = -206$$

Minus sign indicates 180° phase shift between the input and output signals.



Transistor Amplifiers MOSFET Amplifier Concept



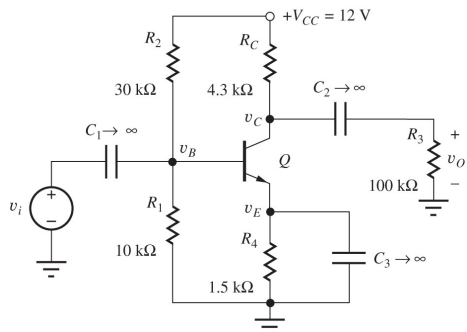
MOSFET is biased in active region by dc voltage source V_{GS} . e.g., Q-point is set at $(I_D, V_{DS}) = (1.56 \text{ mA}, 4.8 \text{ V})$ with $V_{GS} = 3.5 \text{ V}$

Total gate-source voltage is: $v_{GS} = V_{GS} + v_{gs}$

1 V_{p-p} change in v_{GS} yields 1.25 mA $_{p-p}$ change in i_D and a 4 V_{p-p} change in v_{DS}



Transistor Amplifiers Coupling and Bypass Capacitors



- ac coupling through capacitors is used to inject ac input signal and extract output signal without disturbing Q-point

- Capacitors are designed to provide negligible impedance at frequencies of interest and provide open circuits at dc.
- C_1 and C_2 are low impedance coupling capacitors or dc blocking capacitors whose reactance at the signal frequency is designed to be negligible.
- C_3 is a bypass capacitor that provides a low impedance path for ac current from emitter to ground, thereby removing R_E (required for good Q-point stability) from the circuit when ac signals are considered.

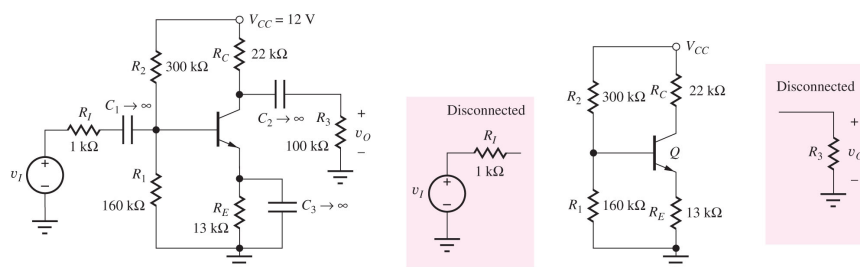


Transistor Amplifiers dc and ac Analysis – Two Step Analysis

- **dc analysis:**
 - Find dc equivalent circuit by replacing all capacitors by open circuits and inductors by short circuits.
 - Find Q-point from dc equivalent circuit by using appropriate large-signal transistor model.
- **ac analysis:**
 - Find ac equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, dc voltage sources by ground connections and dc current sources by open circuits.
 - Replace transistor by its small-signal model
 - Use small-signal ac equivalent to analyze ac characteristics of amplifier.
 - Combine end results of dc and ac analysis to yield total voltages and currents in the network.



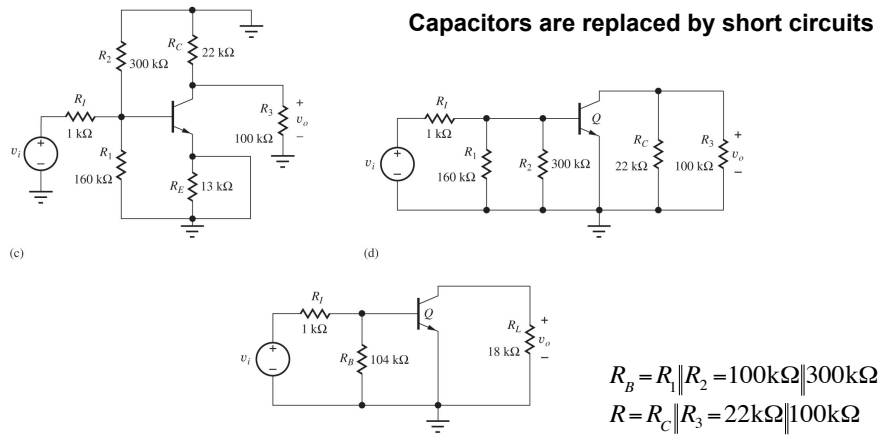
Transistor Amplifiers dc Equivalent Circuit for BJT Amplifier



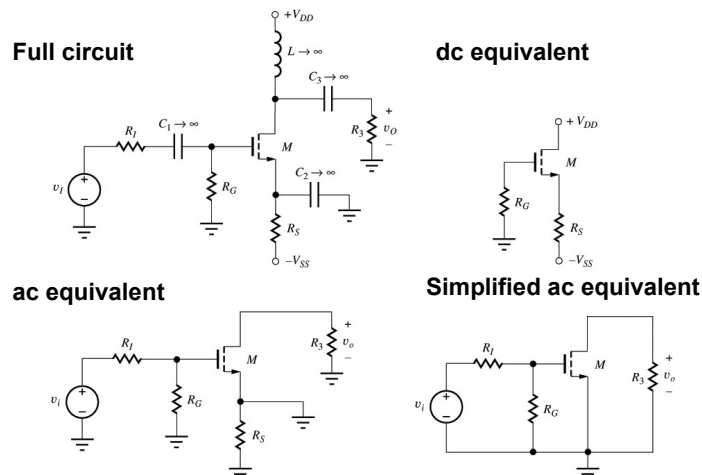
- All capacitors in the original amplifier circuit are replaced by open circuits, disconnecting v_i , R_F , and R_3 from circuit.



Transistor Amplifiers ac Equivalent Circuit for BJT Amplifier

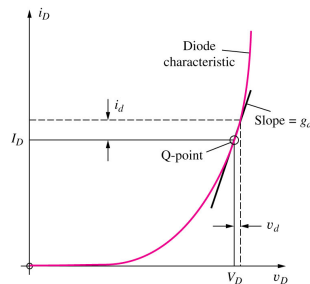


Transistor Amplifiers dc and ac Equivalents for a MOSFET Amplifier



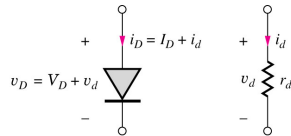
Small-Signal Operation Diode Small-Signal Model

- The slope of the diode characteristic at the Q-point is called the diode conductance and is given by:



$$g_d = \left. \frac{\partial i_D}{\partial v_D} \right|_{Q\text{-point}} = \frac{I_S}{V_T} \exp\left(\frac{V_D}{V_T}\right) \approx \frac{I_D}{V_T}$$

- Diode resistance is given by:



$$r_d = \frac{1}{g_d}$$

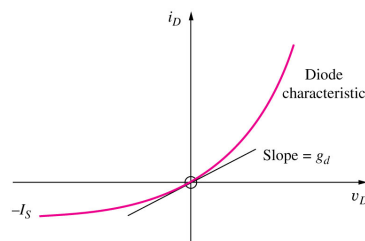


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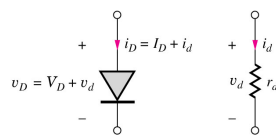


Small-Signal Operation Diode Small-Signal Model (cont.)



- g_d is small but non-zero for $I_D = 0$ because slope of diode equation is nonzero at the origin.
- At the origin, the diode conductance and resistance are given by:

$$g_d = \frac{I_S}{V_T} \quad \text{and} \quad r_d = \frac{V_T}{I_S}$$

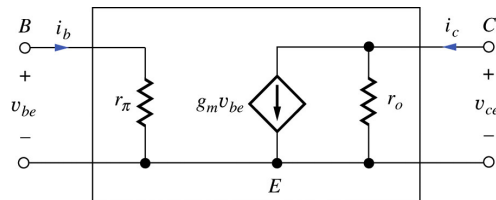


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Small-Signal Operation BJT Hybrid-Pi Model



Transconductance:

$$g_m = \frac{I_C}{V_T} \approx 40 I_C$$

Input resistance:

$$r_\pi = \frac{\beta_o V_T}{I_C} = \frac{\beta_o}{g_m} \quad \text{or} \quad \beta_o = g_m r_\pi$$

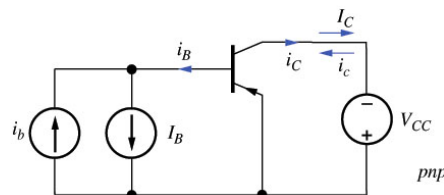
Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$

- The hybrid-pi small-signal model is the intrinsic representation of the BJT.
- Small-signal parameters are controlled by the Q-point and are independent of geometry of the BJT



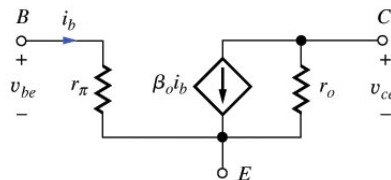
BJT Small-Signal Operation Small-Signal Model for pnp Transistor



- For the pnp transistor

$$i_B = I_B - i_b$$

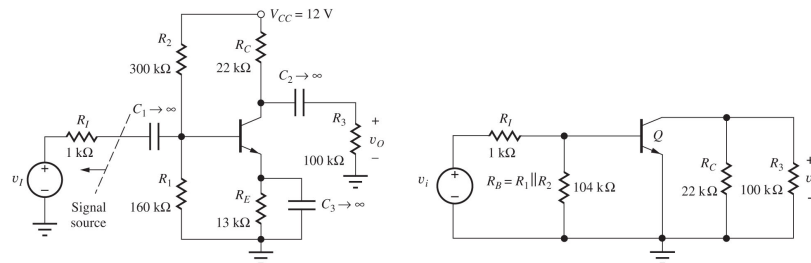
$$i_C = I_C - i_c = \beta_F I_B - \beta_F i_b$$



- Signal current injected into base causes decrease in total collector current which is equivalent to increase in signal current entering collector.
- So the small-signal models for the npn and pnp devices are identical!



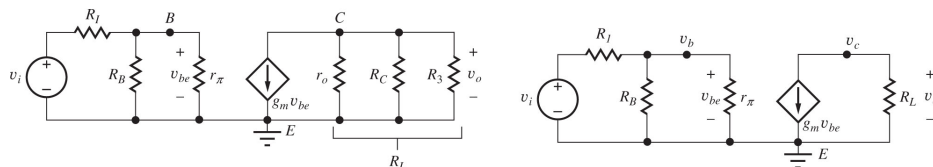
Common-Emitter Amplifiers Small-Signal Analysis - ac Equivalent Circuit



- ac equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources are ac ground.
- Assume that Q-point is already known.



Common-Emitter Amplifiers Small-Signal Equivalent Circuit

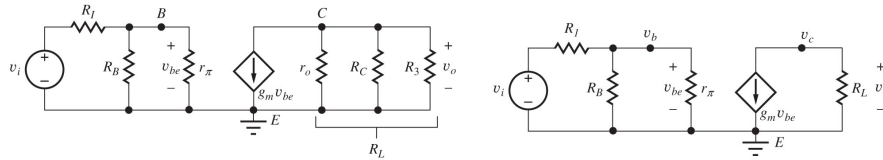


- Input voltage is applied to the base terminal
- Output signal appears at collector terminal
- Emitter is **common** to both input and output signals
Thus circuit is termed a Common-Emitter (C-E) Amplifier.
- The terminal gain of the C-E amplifier is the gain from the base terminal to the collector terminal

$$A_{vt}^{CE} = \frac{v_c}{v_b} = -g_m R_L \quad R_L = r_o \parallel R_C \parallel R_3$$



Common-Emitter Amplifiers Input Resistance and Signal Source Gain



Define R_{iB} as the input resistance looking into the base of the transistor:

$$R_{iB} = \frac{v_b}{i_b} = r_\pi$$

The input resistance presented to v_i is:

$$R_{in} = R_I + R_B \parallel R_{iB} = R_I + R_B \parallel r_\pi$$

The signal source voltage gain is:

$$A_v^{CE} = \frac{v_o}{v_i} = \frac{v_o}{v_b} \frac{v_b}{v_i} = A_{vt}^{CE} \frac{R_B \parallel r_\pi}{R_I + R_B \parallel r_\pi}$$



Common-Emitter Amplifiers “Rule of Thumb” Design Estimate

$$A_v^{CE} = A_{vt}^{CE} \frac{R_B \parallel r_\pi}{R_I + R_B \parallel r_\pi} \cong A_{vt}^{CE} \quad A_{vt}^{CE} = -g_m R_L \quad R_L = r_o \parallel R_C \parallel R_3$$

$$\text{Typically: } r_o \gg R_C \text{ and } R_3 \gg R_C \quad A_v^{CE} \cong -g_m R_C = -40 I_C R_C$$

$I_C R_C$ represents the voltage dropped across collector resistor R_C

$$\text{A typical design point is } I_C R_C = \frac{V_{CC}}{3}$$

$$\therefore A_v^{CE} \cong -40 \frac{V_{CC}}{3} = -13.3 V_{CC}$$

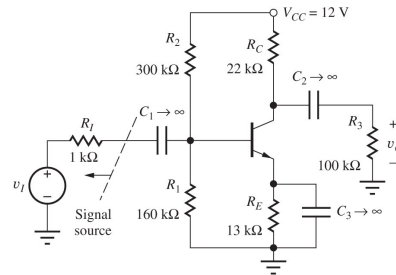
To help account for all the approximations and have a number that is easy to remember, our "rule-of-thumb" estimate for the voltage gain becomes

$$A_v^{CE} \cong -10 V_{CC}$$



Common-Emitter Amplifiers Voltage Gain Example

- **Problem:** Calculate voltage gain, input resistance and maximum input signal level for a common-emitter amplifier
- **Given data:** $\beta_F = 100$, $V_A = 75\text{ V}$, Q-point is (0.245 mA, 3.39 V)
- **Assumptions:** Transistor is in active region, $\beta_o = \beta_F$. Signals are low enough to be considered small signals. Room temperature.



- **Analysis:**

$$g_m = 40I_C = 40(0.245\text{ mA}) = 9.80\text{ mS} \quad r_\pi = \frac{\beta_o}{g_m} = \frac{100}{9.8\text{ mS}} = 10.2\text{ k}\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{75\text{ V} + 3.39\text{ V}}{0.245\text{ mA}} = 320\text{ k}\Omega \quad R_B = R_1 \parallel R_2 = 160\text{ k}\Omega \parallel 300\text{ k}\Omega = 104\text{ k}\Omega$$

$$R_L = r_o \parallel R_C \parallel R_L = 320\text{ k}\Omega \parallel 22\text{ k}\Omega \parallel 100\text{ k}\Omega = 17.1\text{ k}\Omega \quad R_B \parallel r_\pi = 104\text{ k}\Omega \parallel 10.2\text{ k}\Omega = 9.29\text{ k}\Omega$$



Common-Emitter Amplifiers Voltage Gain Example (cont.)

$$A_v = -g_m R_L \left(\frac{R_B \parallel r_\pi}{R_I + R_B \parallel r_\pi} \right) = -9.8\text{ mS} (17.1\text{ k}\Omega) \left(\frac{9.29\text{ k}\Omega}{1\text{ k}\Omega + 9.29\text{ k}\Omega} \right) = -168(0.903) = -151$$

$$R_{in} = R_I + R_B \parallel r_\pi = 10.3\text{ k}\Omega$$

$$v_{be} = v_i \left(\frac{R_B \parallel r_\pi}{R_I + R_B \parallel r_\pi} \right) \quad \therefore v_{be} \leq 0.005\text{ V} \rightarrow v_i \leq 5\text{ mV} \left(\frac{10.3\text{ k}\Omega}{9.29\text{ k}\Omega} \right) = 5.54\text{ mV}$$

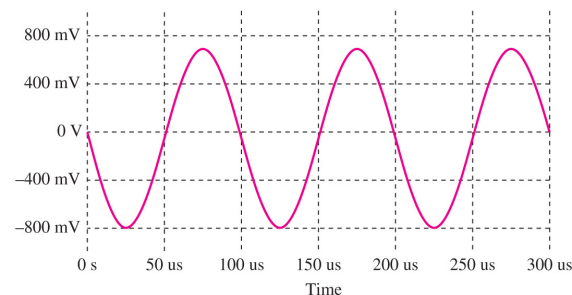
Check the rule-of-thumb estimate: $A_v^{CE} \approx -10(12) = -120$ (ballpark estimate)

What is the maximum amplitude of the output signal: $v_o \leq 5.54\text{ mV}(-151) = 0.837\text{ V}$

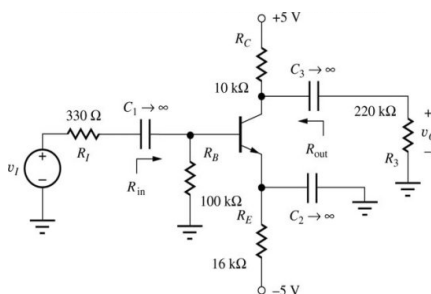


Common-Emitter Amplifiers Voltage Gain Example (cont.)

- **Simulation Results:** The graph below presents the output voltage for an input voltage that is a 5-mV, 10-kHz sine wave.
- Note that although the sine wave at first looks good, the positive and negative peak amplitudes are different indicating the presence of distortion. The input is near our small-signal limit for linear operation.



Common-Emitter Amplifiers Dual Supply Operation - Example



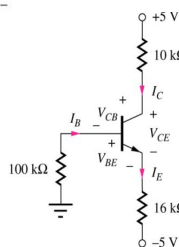
Analysis: To find the Q-point, the dc equivalent circuit is constructed.

$$10^5 I_B + V_{BE} + (\beta_F + 1) I_B (1.6 \times 10^4) = 5$$

$$\therefore I_B = 3.71 \mu A$$

$$I_C = 65 I_B = 241 \mu A$$

$$I_E = 66 I_B = 245 \mu A$$



$$5 - 10^4 I_C - V_{CE} - (1.6 \times 10^4) I_E - (-5) = 0$$

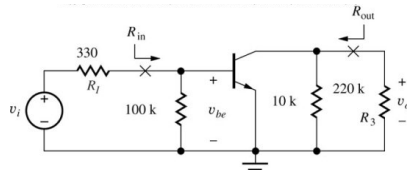
$$\therefore V_{CE} = 3.67 V$$

- **Problem:** Find voltage gain, input and output resistances for the circuit above
- **Given:** $\beta_F = 65$, $V_A = 50 V$
- **Assumptions:** Active-region operation, $V_{BE} = 0.7 V$, small signal operating conditions.



Common-Emitter Amplifiers Dual Supply Operation - Example (cont.)

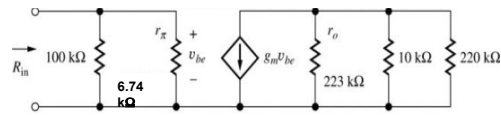
- Next we construct the ac equivalent and simplify it.



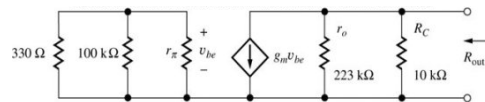
$$g_m = 40I_C = 9.64 \times 10^{-3} \text{ S}$$

$$r_\pi = \frac{\beta_o}{40I_C} = 6.74 \text{ k}\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = 223 \text{ k}\Omega$$



$$R_{in} = R_B \parallel r_\pi = 6.31 \text{ k}\Omega$$



$$R_{out} = R_C \parallel r_o = 9.57 \text{ k}\Omega$$

$$A_v^{CE} = \frac{v_o}{v_i} = -g_m (R_{out} \parallel R_3) \left(\frac{R_{in}}{R_f + R_{in}} \right) = -84.0$$

$$\text{Gain Estimate: } A_v^{CE} \cong -10(V_{CC} + V_{EE}) = -100$$

